Unit 5 Review Quesons

1. Skydivers jump out of an airplane at an altude of 3.5 km. The equaon H = 3500 - 5t² models the altude, H, in metres, of the skydivers, at t seconds aer jumping out of the airplane.
a) How far have the skydivers fallen aer 10 s?
b) The skydivers open their parachutes at an altude of 1000m. How long did they free fall?

2. Kae sells specialty teddy bears at various summer fesvals. Her profit for a week , P, in dollars, can be modelled by P = -0.1n² + 30n - 1200, where n is the number of teddy bears she sells during the week.
a) How many teddy bears would she have to sell to earn $500?
b) How many teddy bears would she have to sell to break even?
c) How many teddy bears would she have to sell to maximize her profit?

3. Determine the dimensions of a rectangle that has a perimeter of 40 cm and has a maximum area. What is the maximum area?

4. A farmer wants to make a rectangular corral along the side of a large barn and has only 60m of fencing. Only 3 sides must be fenced, since the barn will form the fourth side. What should the dimensions of the corral be in order to enclose the maximum area?

5. Find two numbers whose sum is 34 and whose product is a maximum.

6. The path of a basketball shot can be modelled by the equaon:

$$h = -0.09d^2 + 0.9d + 2$$

where h is the height of the basketball in metres and d is the horizontal distance of the ball from the player in metres. What is the maximum height reached by the ball?

7. Give an example of a quadrac equaon with
a) no real roots b) one real root c) two real roots

Review in Text
Pg. 316 # 1-11
Pg. 318 #1-16

Reviewing tests #3 and #4 would also be a good idea!

1. Skydivers jump out of an airplane at an altitude of 3.5 km. The equation $H = 3500 - 5t^2$ models the altitude, H, in metres, of the skydivers, at t seconds after jumping out of the airplane.
a) How far have the skydivers fallen after 10 s?
b) The skydivers open their parachutes at an altitude of 1000m. How long did they free fall?

Soluon to #1:

a) $H = 3500 - 5(10)^2$
$= 3500 - 500$
$= 3000$ m ∴ They have fallen 500m in 10 seconds

b) Let $H = 1000$
$1000 = 3500 - 5t^2$
$5t^2 = 2500$
$t^2 = 500$
$t = \sqrt{500}$ (t cannot be Θive)
$t \doteq 22.4$ seconds ∴ They are freefalling for 22.4 seconds

2. Katie sells specialty teddy bears at various summer festivals. Her profit for a week , P, in dollars, can be modelled by $P = -0.1n^2 + 30n - 1200$, where n is the number of teddy bears she sells during the week.
a) How many teddy bears would she have to sell to earn $500?
b) How many teddy bears would she have to sell to break even?
c) How many teddy bears would she have to sell to maximize her profit?

Soluon to #2:

a) Maximum Profit

$$P = -0.1(n^2 - 300n) - 1200$$
$$P = -0.1(n^2 - 300n + 22500 - 22500) - 1200$$
$$P = -0.1(n - 150)^2 + 2250 - 1200$$
$$P = -0.1(n - 150)^2 + 1050$$

Maximum profit in one week is $1050 so no.

b) let P = 500

$$-0.1n^2 + 30n - 1200 = 500$$
$$-0.1n^2 + 30n - 1700 = 0$$
$$n = \frac{-30 \pm \sqrt{30^2 - 4(-0.1)(-1700)}}{2(-0.1)}$$
$$n = \frac{-30 \pm \sqrt{220}}{-0.2}$$
$$n \doteq 75.8 \text{ or } n \doteq 224.2$$

c) let P = 0

$$-0.1n^2 + 30n - 1200 = 0$$
$$n = \frac{-30 \pm \sqrt{30^2 - 4(-0.1)(-1200)}}{2(-0.1)}$$
$$n = \frac{-30 \pm \sqrt{420}}{-0.2}$$
$$n \doteq 47.5 \text{ or } n \doteq 252.5$$

d) 150 (see part a)

3. Determine the dimensions of a rectangle that has a perimeter of 40 cm and has a maximum area. What is the maximum area?

Soluon to #3:

① $40 = w + w + L + L$

$40 = 2w + 2L$

$\frac{-2L}{-2} = \frac{2w - 40}{-2}$

$L = -w + 20$

② $A = w(-w + 20)$

$= -w^2 + 20w$

$= -(w^2 + 20w)$

$= -(w^2 - 20w + 100) + 100$

$= -(w-10)^2 + 100$

∴ The maximum area is $100 cm^2$.

4. A farmer wants to make a rectangular corral along the side of a large barn and has only 60m of fencing. Only 3 sides must be fenced, since the barn will form the fourth side. What should the dimensions of the corral be in order to enclose the maximum area?

Soluon to #4:

① 

$2w + L = 60$

$2w - 60 = -L$

$-2w + 60 = L$

② $A = w(-2w + 60)$

$A = -2w^2 + 60w$

$A = -2(w^2 - 30w)$

$A = -2(w^2 - 30w + 225 - 225)$

$A = -2(w - 15) + 450$

③ (15, 450)

④ $w = 15$

$A = 450$

⑤ $L = -2w + 60$

$L = -2(15) + 60$

$L = 30$

5. Find two numbers whose sum is 34 and whose product is a maximum.

Let x be one number

$\therefore$ other # is $(34-x)$

Product $= (x)(34-x)$ $\Rightarrow$ need vertex

Zeros at $x=0$ & $x=34$

$\therefore$ Vertex : $x = \frac{0+34}{2}$

$x = 17$

$y = (17)(34-17)$

$= 289$

$\therefore$ The #s are 17 & 17.

6. The path of a basketball shot can be modelled by the equation:

$$h = -0.09d^2 + 0.9d + 2$$

where h is the height of the basketball in metres and d is the horizontal distance of the ball from the player in metres. What is the maximum height reached by the ball?

* Need vertex $\rightarrow$ complete the square

$h = -0.09d^2 + 0.9d + 2$

$= -0.09(d^2 - 10d) + 2$

$= -0.09(d^2 - 10d + 25 - 25) + 2$

$= -0.09(d-5)^2 + 2.25 + 2$

$= -0.09(d-5)^2 + 4.25$

$\therefore$ The ball reaches a max. height of 4.25 m.

7. Give an example of a quadratic equation with

a) no real roots

$a \cdot k > 0$

opens up & vertex above x-axis

OR

opens down & vertex below x-axis

b) one real root

$k = 0$

vertex is on the x-axis

c) two real roots

$a \cdot k < 0$

opens up & vertex below x-axis

OR

opens down & vertex above x-axis