Completing the square involves taking a quadratic relation of the form *$y = ax^2 + bx + c$* into vertex form *$y=a(x-h)^2+k$*

Ex. 3 Write each of the following in vertex form.

a) $y = 5x^2 - 30x + 12$

$$y = 5(x^2 - 6x + 9 - 9) + 12$$

$$\frac{6}{2} = 3 \rightarrow 3^2$$

$$(-9)(5)$$

$$y = 5(x^2 - 6x + 9) - 45 + 12$$

$$y = 5(x-3)^2 - 33$$

b) $y = -3x^2 - 12x + 5$

$$y = -3(x^2 + 4x + 4 - 4) + 5$$

$$\left(\frac{4}{2}\right)^2$$

$$y = -3(x^2 + 4x + 4) + 12 + 5$$

$$y = -3(x+2)^2 + 17$$

c) $y = -2x^2 + 16x - 3$

$$y = -2(x^2 - 8x \qquad\qquad) - 3$$

$$y = -2(x^2 - 8x + 16 - 16) - 3$$

$$y = -2(x^2 - 8x + 16) + 32 - 3$$

$$y = -2(x-4)^2 + 29$$

Ex. 4 Determine the maximum or minimum point of each parabola.

a. $y = 2x^2 + 8x + 7$

$y = 2(x^2 + 4x + 4 - 4) + 7$

$y = 2(x^2 + 4x + 4) - 8 + 7$

$y = 2(x+2)^2 - 1$

$\therefore$ Min is $(-2, -1)$

? $a > 0$

b. $y = x^2 - 14x + 20$

$y = x^2 - 14x + 49 - 49 + 20$

$y = (x-7)^2 - 29$

$\therefore$ Min is $(7, -29)$

c. $y = -3x^2 - 12x + 5$

$y = -3(x^2 + 4x + 4 - 4) + 5$

$y = -3(x^2 + 4x + 4) + 12 + 5$

$y = -3(x+2)^2 + 17$

$\therefore$ Max is $(-2, 17)$

? $a < 0$

d. $y = 5x^2 + 10x - 11$

$y = 5(x^2 + 2x + 1 - 1) - 11$

$y = 5(x^2 + 2x + 1) - 5 - 11$

$y = 5(x+1)^2 - 16$

$\therefore$ Min is $(-1, -16)$

e. $y = -4x^2 + 24x - 3$

$y = -4(x^2 - 6x + 9 - 9) - 3$

$y = -4(x^2 - 6x + 9) + 36 - 3$

$y = -4(x-3)^2 + 33$

$\therefore$ Max is $(3, 33)$

f. $y = 3x^2 - 9x + 4$

$y = 3(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) + 4$

$\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

$y = 3(x^2 - 3x + \frac{9}{4}) - \frac{27}{4} + 4$

$y = 3(x - \frac{3}{2})^2 - \frac{27}{4} + \frac{16}{4}$

$y = 3(x - \frac{3}{2})^2 - \frac{11}{4}$

$\therefore$ Min is $\left(\frac{3}{2}, -\frac{11}{4}\right)$

Ex. 1 The path of a basketball shot can be modelled by the equation $h = -0.09d^2 + 0.9d + 2$ where h is the height of the basketball in metres and d is the horizontal distance of the ball from the player in metres.

a. What is the maximum height reached by the ball?

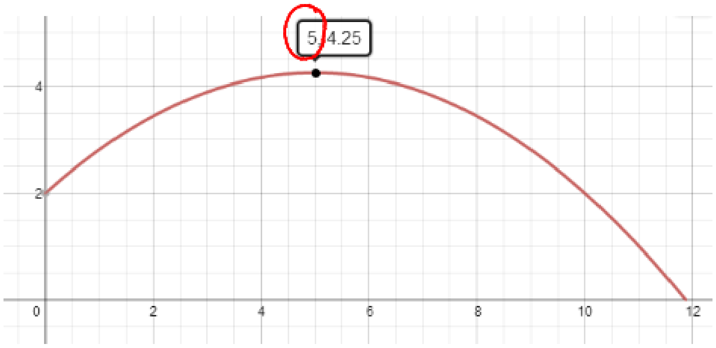
$$h = -0.09d^2 + 0.9d + 2$$
$$h = -0.09(d^2 - 10d + 25 - 25) + 2$$
$$h = -0.09(d^2 - 10d + 25) + 2.25 + 2$$
$$h = -0.09(d-5)^2 + 4.25$$

Vertex (5, 4.25)

Max height

∴ Maximum height was 4.25m

b. How far is the ball from the player when it reaches maximum height?



The ball is 5m from the player.