11. Write an equation for the larger circle.

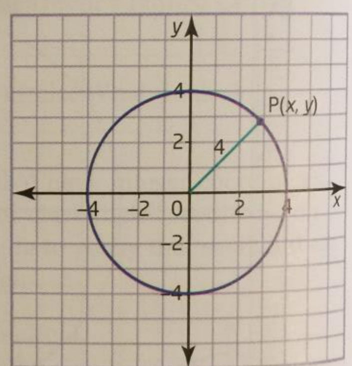
12. Reflect Write an equation for the circle with centre (0, 0) and radius r. Then, use this equation to write an expression for the radius.

Example 1 Equation for a Circle

Find an equation for the circle with centre (0, 0) and radius 4.

Solution

The distance from the origin to any point P(x, y) on the circle is the length of the radius. So,



OP = 4

The distance formula also gives an expression for the length of OP:

$$\begin{aligned} OP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x - 0)^2 + (y - 0)^2} \\ &= \sqrt{x^2 + y^2} \end{aligned}$$

Therefore,

$$\sqrt{x^2 + y^2} = 4$$

$$x^2 + y^2 = 16$$

An equation for the circle is $x^2 + y^2 = 16$.

Example 2 Determine Whether a Point Lies Within a Circle

a) Determine an equation and the radius for the circle that has its centre at the origin and passes through the point A(6, −8).

b) Is the point B(−5, 9) inside this circle?

Solution

a) An equation for a circle centred at the origin has the form $x^2 + y^2 = r^2$.

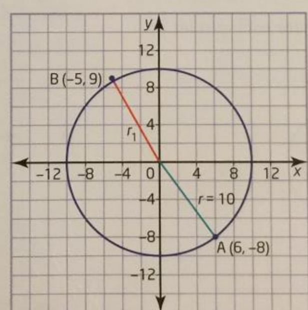
Substitute the coordinates of the point (6, −8) into the equation for the circle.

The point (6, −8) lies on this circle, so the coordinates of the point must satisfy the equation of the circle.

$$x^2 + y^2 = r^2$$
$$6^2 + (-8)^2 = r^2$$
$$36 + 64 = r^2$$
$$100 = r^2$$
$$\sqrt{100} = \sqrt{r^2}$$
$$10 = r$$

An equation for the circle is $x^2 + y^2 = 100$, and the radius of the circle is 10.

b) Consider a circle with its centre at the origin and with point B(−5, 9) on the circumference. Let r_1 be the radius of this circle. To find the length of the radius, substitute the coordinates of point B into the formula for the radius of a circle centred at the origin.



$$r_1 = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-5)^2 + 9^2}$$
$$= \sqrt{25 + 81}$$
$$= \sqrt{106}$$
$$\doteq 10.3$$

Since $r_1 > 10$, point B lies outside the circle defined by $x^2 + y^2 = 100$.

If $r_1 > r$, then $r_1^2 > r^2$. So, the inequality $x^2 + y^2 > r^2$ defines the region *outside* the circle with centre (0, 0) and radius r.

Reflect