1.6 Solving by Elimination

What were the big ideas from yesterday's class?

- An equivalent linear system is formed by multiplying/dividing any of the equations by a constant.

- An equivalent linear system is formed by adding or subtracting the equations.

Eliminate means to <u>remove</u> or get rid of.
What do you think we would like to eliminate?

An unknown!

Consider the following system:

$$\begin{array}{rl} & x - y = 1 \\ + & 3x + y = 11 \\ \hline & 4x \quad = 12 \\ & x = 3 \end{array}$$

What happens when we add the equations?

Why? Signs were different
Coefficients are same

To add or to subtract?
Think...

$$\begin{array}{rc} - & y \\ & y \\ \hline & 0y \end{array} \qquad \begin{array}{rc} + & -y \\ & y \\ \hline & 0y \end{array} \qquad \begin{array}{rc} + & y \\ & -y \\ \hline & 0y \end{array} \qquad \begin{array}{rc} - & -y \\ & -y \\ \hline & 0y \end{array}$$

Opposite signs --> add
Same signs --> subtract

Examples: Solve by elimination.

a. $x - y = 1$ ①
$3x + y = 11$ ②

① + ② $4x = 12$
$x = 3$

$\therefore$ Sol$^{\underline{n}}$ $(3, 2)$

Sub $x=3$ into ①
$(3) - y = 1$
$-y = -2$
$y = 2$

METHOD 2 - THE ELIMINATION METHOD

1. Multiply one or both equations by a constant so that the coefficients of either *x* or *y* are the same in both equations (sign does not matter).
2. Add or subtract the equations to *eliminate* one variable.
3. Solve the remaining equation.
4. Substitute the solved value into one of the original equations to determine the value of the other variable.
5. Write a conclusion.
6. Check (formally if asked, otherwise mentally.

b. $x + 3y = 2$ ①
$2x + 5y = 3$ ②

① × 2 $\quad 2x + 6y = 4$
② $\quad 2x + 5y = 3$
① − ② $\quad y = 1$

Sub $y = 1$ into ①
$x + 3(1) = 2$
$x = -1$

$\therefore$ Sol^{n} $(-1, 1)$

c. $5x - 3y = 9$ ①
$2x - 5y = -4$ ②

① × 2 $\quad 10x - 6y = 18$
② × 5 $\quad 10x - 25y = -20$
① − ② $\quad 19y = 38$
$y = 2$

Sub $y = 2$ into ②
$2x - 5(2) = -4$
$2x = -4 + 10$
$2x = 6$
$x = 3$

$\therefore$ Sol^{n} $(3, 2)$

d. $2x + 3y = 8$ ①
$3x - 5y = 2$ ②

①×3 $6x + 9y = 24$
②×2 $6x - 10y = 4$
①−② $19y = 20$
$y = \frac{20}{19}$

Sub into ①
$2x + 3\left(\frac{20}{19}\right) = 8$
$2x + \frac{60}{19} = 8$
$2x = 8 - \frac{60}{19}$

Approach 1
$(\quad\quad) \times 19$
$38x = 152 - 60$
$38x = 92$
$x = \frac{92}{38}$
$= \frac{46}{19}$

Approach 2
$2x = 8 - \frac{60}{19}$
$2x = \frac{152}{19} - \frac{60}{19}$
$2x = \frac{92}{19}$
$x = \frac{92}{38}$
$= \frac{46}{19}$

∴ Sol^{n} $\left(\frac{46}{19}, \frac{20}{19}\right)$

e. $3m = -1 - 4n$
$5n = 4m + 22$

Need same order!

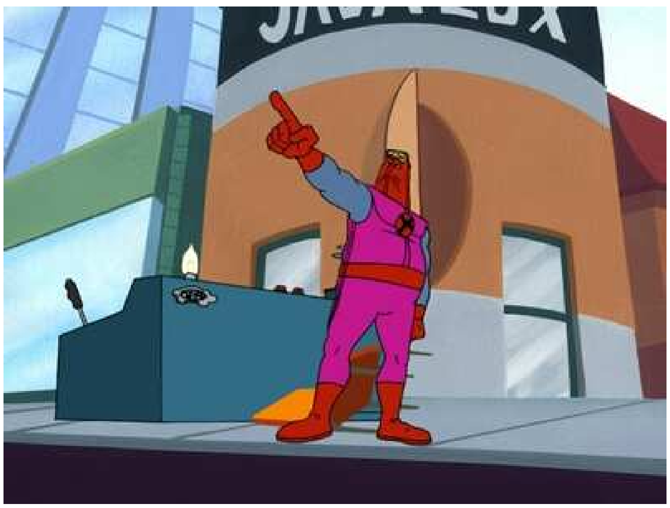
⇒ $3m = -1 - 4n$ ①
$-4m = 22 - 5n$ ②

①×4 $12m = -4 - 16n$
②×3 $-12m = 66 - 15n$
①+② $0 = 62 - 31n$
$31n = 62$
$n = 2$

Sub into ①
$3m = -1 - 4(2)$
$3m = -9$
$m = -3$

∴ Sol^{n}
$m = -3$
$n = 2$

Homework



The Eliminator

... solving systems by eliminating one variable at a time!

p.40 # 2d,3d,5d,7cd,10,12c,13,18,19b