Example 4 A Fish Tale

Stephanie has five more fish in her aquarium than Brett has. The two have a total of 31 fish. How many fish does Stephanie have? How many fish does Brett have?

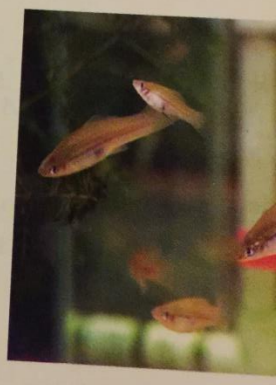
Solution

Model the information using equations.
Let S represent the number of fish that Stephanie has.
Let B represent the number of fish that Brett has.

From the first sentence,
$S = 5 + B$ ①

From the second sentence,
$S + B = 31$ ②

I have two linear equations in two unknowns, so this is a linear system.



Substitute $5 + B$ for S in ②.

$$\begin{aligned} S + B &= 31 \\ 5 + B + B &= 31 \\ 5 + 2B &= 31 \\ 2B &= 31 - 5 \\ 2B &= 26 \\ B &= 13 \end{aligned}$$

Substitute 13 for B in ①.

$$\begin{aligned} S &= 5 + B \\ S &= 5 + 13 \\ S &= 18 \end{aligned}$$

Look back: Verify that this solution works in the original problem statements. Stephanie has 5 more fish than Brett has. 18 is 5 more than 13. The two have 31 fish altogether. $18 + 13 = 31$.
Make a final statement: Stephanie has 18 fish and Brett has 13 fish.

L.S.
$= 4(1) + 3(3)$
$= 4 + 9$
$= 13$
L.S. = R.S.

R.S.
$= 5 - 12$
$= -7$
L.S. = R.S.

The point of intersection of the lines is $(1, 3)$.

Example 4 Solve a Problem Using the Method of Elimination

A small store sells used CDs and DVDs. The CDs sell for \$9 each. The DVDs sell for \$11 each. Cody is working part time and sells a total of \$204 worth of CDs and DVDs during his shift. He knows that 20 items were sold. He needs to tell the store owner how many of each type were sold. How many CDs did Cody sell? How many DVDs did Cody sell?

Solution

Let c represent the number of CDs sold.
Let d represent the number of DVDs sold.

$c + d = 20$ ①
$9c + 11d = 204$ ②

The number of CDs plus the number of DVDs is 20.

\$9 for each CD plus \$11 for each DVD totals \$204.

I can also solve this system using substitution or graphing.

If I subtract, c is eliminated.

Multiply ① by 9.
$9c + 9d = 180$
$9c + 11d = 204$
$-2d = -24$
$d = 12$

Substitute $d = 12$ into one of the original equations to solve for c.
$c + d = 20$
$c + 12 = 20$
$c = 8$

Check in the original word problem:
Money: 8 CDs at \$9 is \$72, and 12 DVDs at \$11 is \$132. The total is \$204.
Number of items: 8 CDs and 12 DVDs is 20 items sold.

Cody sold eight CDs and twelve DVDs during his shift.

choose to solve by elimination.

Example 1 Graphing, Substitution, or Elimination?

Christian has a total of eight cars and trucks to play with. His birthday is soon. He hopes to double the number of cars he has now. If he does, he will have a total of 11 cars and trucks. How many cars does he have now? How many trucks?

Solution

Let c represent the number of cars Christian has now.
Let t represent the number of trucks he has now.
$c + t = 8$
$2c + t = 11$

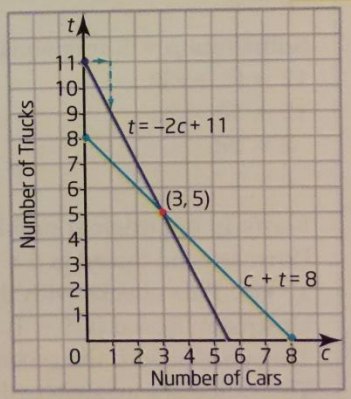
For the line $c + t = 8$, the intercepts are at (8, 0) and (0, 8).
Rearrange the second equation as $t = -2c + 11$.
The t-intercept is 11 and the slope is -2.

The solution is $c = 3$, $t = 5$.

Check in the problem:

Christian's cars and trucks now:
3 cars + 5 trucks = 8 toys

Christian's cars and trucks after his birthday: 6 cars + 5 trucks = 11 toys
Also, 6 cars is double 3 cars.
This checks.
Christian has three cars and five trucks now.



It doesn't m
of a car, so
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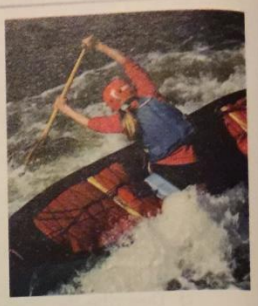
The problem in Example 1 was solved by graphing, but it can be solved by any of the three methods: graphing, substitution, or elimination.

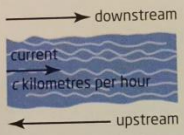
Example 2 Solve a Distance, Speed, Time Problem

A canoeist took 2 h to travel 12 km down a river. The return trip, against the current, took 3 h. What was the average paddling rate of the canoeist? What was the speed of the current?



Solution

Let p represent the canoeist's average paddling speed, in kilometres per hour. Let c represent the speed of the current, in kilometres per hour. Draw a diagram to model the situation. Then, use a table to organize the given facts.



Going downstream, the current helps the canoeist. Going upstream, the current slows the canoeist down.

Direction	Distance (km)	Speed (km/h)	Time (h)
Downstream	12	$p + c$	2
Upstream	12	$p - c$	3

To write the equations, use the fact that distance = speed × time.

$12 = (p + c)2$ ①
$12 = (p - c)3$ ②

I can simplify each equation by dividing both sides of the first equation by 2, and both sides of the second equation by 3.

$$\begin{aligned} 6 &= p + c \\ 4 &= p - c \\ \hline 10 &= 2p \\ p &= 5 \end{aligned}$$

I can solve this linear system directly using elimination. I will add.

Substitute $p = 5$ into equation ① to find c.

$12 = (5 + c)2$
$12 = 10 + 2c$
$2 = 2c$
$c = 1$

Verify in the original problem:
Downstream: Speed is 5 + 1, or 6 km/h. So, in 2 h the distance is 12 km. This checks with the first sentence.
Upstream: Speed is 5 − 1, or 4 km/h. So, in 3 h the distance is 12 km. This checks with the second sentence.
The canoeist's average paddling rate was 5 km/h. The speed of the current was 1 km/h.