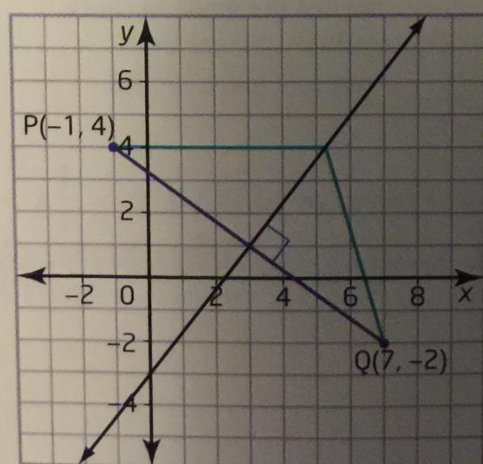
Example 3 Equation of a Right Bisector

Two schools are located at the points P(−1, 4) and Q(7, −2) on a town map. The school board is planning a new sports complex to be used by both schools. The board wants to find a location **equidistant** from the two schools. Use an equation to represent the possible locations for the sports complex.

Solution

From the diagram, you can see that a point can be the same distance from both schools without being directly between them. In fact, any point on the **right bisector** of a line segment is equidistant from the endpoints of the segment. The possible locations for the athletic complex lie on the right bisector of PQ.



To determine an equation for the right bisector, find the slope of the bisector and the coordinates of the midpoint of PQ. First, determine the slope of PQ.

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - (-2)}{-1 - 7}$$
$$= \frac{6}{-8}$$
$$= -\frac{3}{4}$$

Perpendicular lines have slopes that are the negative reciprocals of each other. So, the slope of any line perpendicular to PQ is

$$m_{\perp} = \frac{4}{3}$$

To find the negative reciprocal of a fraction, invert the fraction and use the opposite sign.

The right bisector passes through the midpoint of PQ. Use the midpoint formula to find the coordinates of the midpoint.

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-1 + 7}{2}, \frac{4 + (-2)}{2}\right)$$
$$= \left(\frac{6}{2}, \frac{2}{2}\right)$$
$$= (3, 1)$$

Now, use the coordinates of the midpoint with the slope to solve for the y-intercept of the right bisector.

$$y = mx + b$$
$$1 = \frac{4}{3}(3) + b$$
$$1 = 4 + b$$
$$1 - 4 = b$$
$$-3 = b$$

I can use the graph to check that this value for the y-intercept is reasonable.

An equation for the right bisector of PQ is $y = \frac{4}{3}x - 3$. This equation represents the possible locations for the sports complex.

Example 2 Compare Distances

An air ambulance service uses a grid system to help estimate flying times and fuel requirements. Coordinates on this grid are distances in kilometres east and north of a reference point on the lower left corner of a map of northern Ontario. A helicopter ambulance picks up a patient at point P(96, 197). The nearest hospitals that can provide the treatment the patient needs are in Timmins at T(200, 296) and Sudbury at S(232, 80).

a) To which hospital should the helicopter take the patient?

b) List any assumptions you made for your answer.

Solution

a) First, find the distance to each hospital.

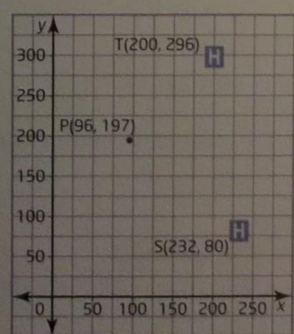
For the Timmins hospital:

$$\begin{aligned} PT &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(200 - 96)^2 + (296 - 197)^2} \\ &= \sqrt{104^2 + 99^2} \\ &= \sqrt{20\,617} \\ &\doteq 144 \end{aligned}$$

For the Sudbury hospital:

$$\begin{aligned} PS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(232 - 96)^2 + (80 - 197)^2} \\ &= \sqrt{136^2 + (-117)^2} \\ &= \sqrt{32\,185} \\ &\doteq 179 \end{aligned}$$

The helicopter should go to the Timmins hospital because it is closer to the pick-up point.



b) The decision to go to the closer hospital assumes that the helicopter can travel in a straight line to either hospital. The decision also assumes that weather will not affect the flying times or prevent a landing at the closer hospital.

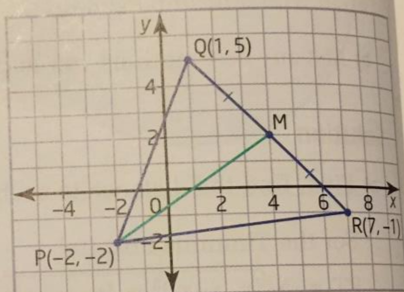
Example 3 Find the Length of a Median

Find the length of the median from P for a triangle with vertices P(−2, −2), Q(7, −1), and R(1, 5).

Solution

The median is the line segment that joins P to the midpoint, M, of QR. To find the coordinates of M, substitute the coordinates of Q and R into the midpoint formula.

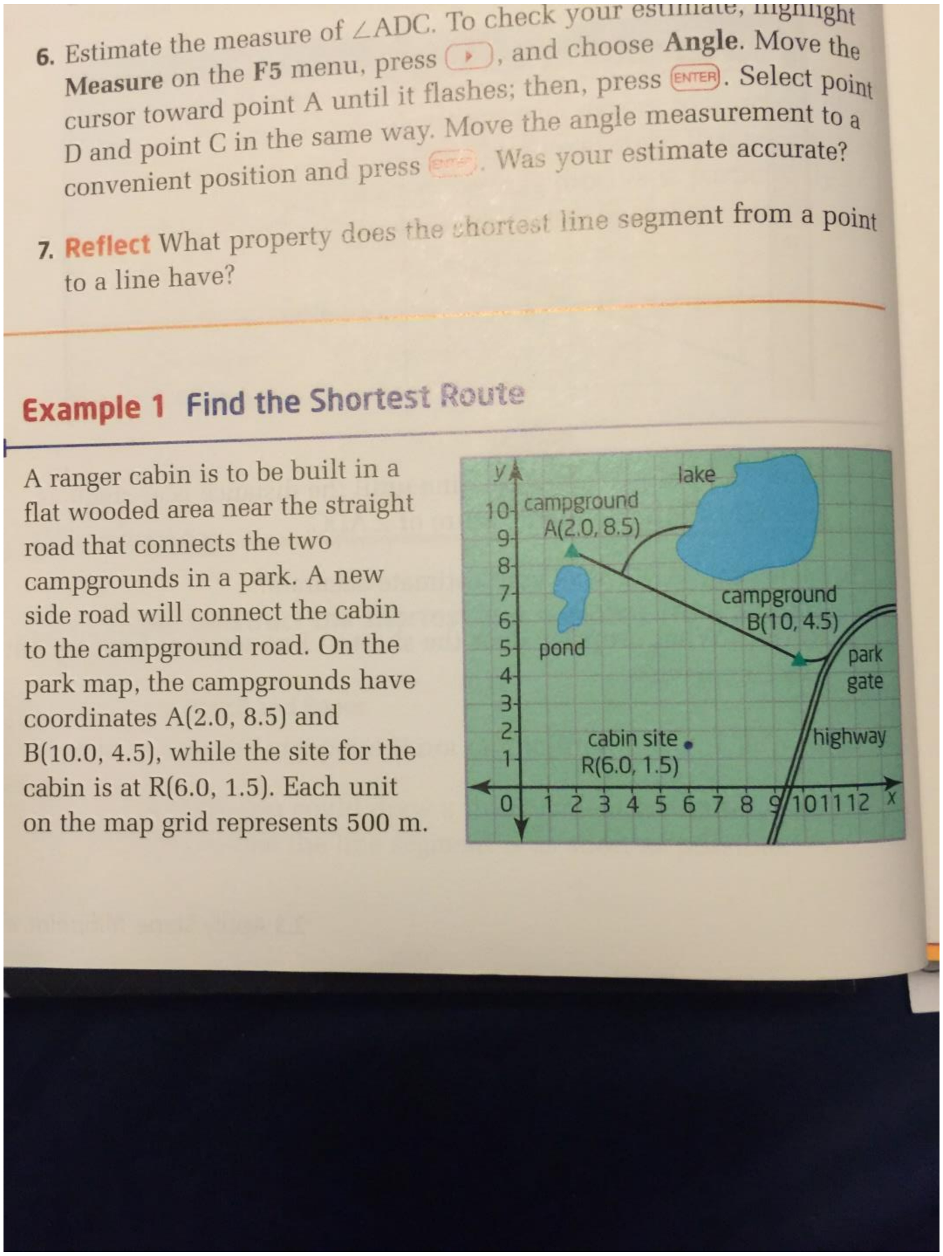
$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{7 + 1}{2}, \frac{-1 + 5}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{4}{2}\right)$$
$$= (4, 2)$$



Now, substitute the coordinates of P(−2, −2) and M(4, 2) into the length formula.

$$PM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[4 - (-2)]^2 + [2 - (-2)]^2}$$
$$= \sqrt{6^2 + 4^2}$$
$$= \sqrt{36 + 16}$$
$$= \sqrt{52}$$

The length of the median from vertex P is $\sqrt{52}$.



6. Estimate the measure of ∠ADC. To check your estimate, highlight **Measure** on the **F5** menu, press [▸], and choose **Angle**. Move the cursor toward point A until it flashes; then, press [ENTER]. Select point D and point C in the same way. Move the angle measurement to a convenient position and press [ENTER]. Was your estimate accurate?

7. **Reflect** What property does the shortest line segment from a point to a line have?

Example 1 Find the Shortest Route

A ranger cabin is to be built in a flat wooded area near the straight road that connects the two campgrounds in a park. A new side road will connect the cabin to the campground road. On the park map, the campgrounds have coordinates A(2.0, 8.5) and B(10.0, 4.5), while the site for the cabin is at R(6.0, 1.5). Each unit on the map grid represents 500 m.

a) Find the route that minimizes the cost and the number of trees that have to be cut down for the side road. Draw a diagram of this route.

b) Find the length of the side road, to the nearest tenth of a kilometre.

Solution

Understand the Problem

Since the area is level, the shortest route for the side road is the cheapest and easiest to build. The shortest route from the ranger cabin to the campground road is perpendicular to that road. To describe the route of the side road and to calculate its length, find the point where a perpendicular from R meets the line segment AB.

Choose a Strategy

Use the coordinates of points A and B to calculate the slope of AB and find an equation for the line through A and B.

The slope of the side road is the negative reciprocal of the slope of AB. Use this slope to determine an equation for the perpendicular line that passes through point R.

Use the equations for the two lines to find the point of intersection, D. Calculate the length of line segment RD from the coordinates of its endpoints. Then, use the map scale to find the length of the side road.

Carry Out the Strategy

a) Calculate the slope of AB using the coordinates of the campgrounds, A(2.0, 8.5) and B(10.0, 4.5).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4.5 - 8.5}{10.0 - 2.0}$$
$$= \frac{-4.0}{8.0}$$
$$= -0.5$$

Since the slope of AB is -0.5, the slope of any line perpendicular to AB is $-\frac{1}{-0.5}$, or 2.

Perpendicular lines have slopes that are negative reciprocals of each other.

Now, find equations for AB and RD by substituting the slope and the coordinates of a point into $y = mx + b$.

For AB, use A(2.0, 8.5):

$y = mx + b$
$8.5 = -0.5(2.0) + b$
$8.5 = -1.0 + b$
$9.5 = b$

For RD, use R(6.0, 1.5):

$y = mx + b$
$1.5 = 2(6.0) + b$
$1.5 = 12.0 + b$
$-10.5 = b$

I can use the coordinates of point B to check the equation for AB.

An equation for AB is $y = -0.5x + 9.5$ and an equation for RD is $y = 2x - 10.5$.